TRIALITY IN QCD AT ZERO AND FINITE TEMPERATURE: A NEW DIRECTION

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ABSTRACT

Discrete symmetries in grand canonical ensembles and in ensembles canonical with respect to triality are investigated. We speculate about the general phase structure of finite temperature gauge theories with discrete $Z(N)$ symmetry. Low and high temperature phases turn out to be different in both ensembles even for infinite systems. It is argued that gauge theories with matter fields in the fundamental representation should be treated in ensembles canonical with respect to triality if one wants to avoid unphysical predictions. Further, we discuss as a physical consequence of such a treatment the impossibility of the existence of metastable phases in the quark-gluon plasma.

The recent interest in studying the $Z(3)$ structure of hot gauge theories [1]-[9] is caused by the strong will to obtain more insight into the phase structure of strongly interacting matter for physical models and for speculations. In the present article we follow the obvious tendency of the above papers to avoid a polemic with each other, to supplement the conventional approach [10] and to develop our own point of view [1, 2]. The guideline of this exploration is to draw the arguments supporting the conclusion of [1, 3] which come from the impressing recent activity on the role of local and global discrete symmetries in gauge theories [11, 12, 13, 14]. Usually, these symmetries appear as a commuting centre of an underlying gauge group. An important conclusion of the above papers is an appearance of a new Higgs phase where the corresponding centre charges are not screened while a gauge symmetry is broken up to a discrete subgroup. Moreover, unlike in the confined phase these charges could, in principle, be detected at long distances via the Aharonov-Bohm effect. In order to make our arguments more clear we have first to review some aspects of the present status of MC analysis of finite temperature QCD.

1 $Z(3)$ symmetry in the grand canonical ensemble description (GCE). Euclidean formulation

In lattice QCD the gluon fields appear in the form of $SU(3)$ matrices $U_{x,\mu}$ which are defined on links (x,μ) of a four-dimensional euclidean lattice. In finite temperature lattice QCD the correlation function $< L(\vec{r}_1) \cdots L^*(\vec{r}_N) >$ of several Polyakov loops

$$L(\vec{r}) = \frac{1}{3} \, \mathrm{Tr} \prod_{t=1}^{N_t} U_0(\vec{r}, t) \tag{1}$$

can be connected [10] with the free energy F of N infinitely heavy quarks q or antiquarks $\bar{q}$ at the corresponding positions $\vec{r}_1, \cdots, \vec{r}_N$ at temperature T relative to the free energy of the vacuum

$$F(q(\vec{r}_1), \cdots, \bar{q}(\vec{r}_N)) = -T \ln < L(\vec{r}_1) \cdots L^*(\vec{r}_N) > . \tag{2}$$

The thermodynamical average $< \cdots >$ is computed using the partition function

$$Z = \int \mathcal{D}[U] \; e^{-S[U]} \tag{3}$$

as a "sum" over all gauge field configurations U. In pure gluonic QCD for most problems the appropriate choice for S is the Wilson action

$$\begin{aligned} S_G[U] &= \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re\;Tr}\; U_{x,\mu\nu}\right), \quad \beta = \frac{6}{g^2}, \\ U_{x,\mu\nu} &= U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}\,, \end{aligned} \tag{4}$$

where the plaquettes $U_{x,\mu\nu}$ are built of four links in the $\mu\nu$-plane of a four dimensional euclidean lattice.

The action $S_G[U]$ is $Z(3)$ symmetric. This means that multiplication of all links in direction $\mu = 0$ in the three dimensional x, y, z-torus with fixed t, e.g. $t = 0$ by a $Z(3)$ element leaves the action invariant. Such a global $Z(3)$ transformation can be regarded as an aperiodic transformation of the centre of the gauge group. A single Polyakov loop $L(\vec{r})$ transforms under $Z(3)$ nontrivially and therefore the distribution of $L(\vec{r})$ values can be used as an indicator for spontaneous breaking of $Z(3)$ symmetry.

Let us now discuss the results of lattice calculations and their common interpretation. Monte-Carlo calculations in pure gauge lattice QCD show usually a characteristic behaviour of the spatial average L of $L(\vec{r})$ during a Monte-Carlo simulation. In the confinement regime (low β) L scatters symmetrically around zero in the complex plane. In the deconfined phase there appear three $Z(3)$ symmetric maxima of the Polyakov loop distribution in $0°$- and $\pm 120°$-directions. The appearance of the three peak structure of the L-distribution in the deconfinement regime close to the phase transition is considered as a demonstration that spontaneous symmetry breaking on a finite lattice can never happen exactly. As the number of tunnelling events between the maxima decreases with increasing β one commonly expects L to be frozen in the thermodynamical limit in one of the $Z(3)$ directions and thus to get spontaneous $Z(3)$ symmetry breaking. Therefore, one may obtain $\arg < L >= 1, \pm\frac{2\pi}{3}$ in the thermodynamical limit. With dynamical fermions the action S contains a fermionic contribution S_F

$$
\begin{aligned}
&S = S_G + S_F, \\
&S_F = \frac{n_F}{4} \sum_{x,x'} \bar{\Psi}_x M_{x,x'} \Psi_{x'},\ M_{x,x'} = D_{x,x'} + m\delta_{x,x'}
\end{aligned} \tag{5}
$$

which breaks $Z(3)$ symmetry explicitly. In the Kogut-Susskind formulation [15] the fermionic matrix M reads

$$
\begin{aligned}
&M_{x,x'} = \frac{1}{2} \sum_{\mu} \left(\Gamma_{x,\mu} U_{x,\mu} \delta_{x',x+\mu} - \Gamma_{x',\mu} U^{\dagger}_{x',\mu} \delta_{x',x-\mu} \right) + m\delta_{x,x'}, \\
&\Gamma_{x,\mu} = (-1)^{x_1 + x_2 + \cdots + x_{\mu-1}},
\end{aligned} \tag{6}
$$

with one-component Grassmann variables Ψ_x and $\bar{\Psi}_x$. The integration over the Grassmann variables in the path integral

$$
Z = \int \mathcal{D}[U, \bar{\Psi}, \Psi]\, e^{-S} \tag{7}
$$

can be performed analytically and leads to the fermionic determinant $\det M$

$$
Z = \int \mathcal{D}[U]\, e^{-S_G[U]} \det M. \tag{8}
$$

It is easy to see that the fermionic action violates $Z(3)$ symmetry [16]. In the usual MC-iterations one finds that in the low temperature phase the expectation value $< L >$ of a Polyakov loop L is a small positive number and in the high temperature phase the three peak structure of the distribution becomes asymmetric. The maximum at 0° is favoured and the two lower maxima at angles of around $\pm 120^\circ$ are symmetric. This fact is usually interpreted as a consequence of the violation of $Z(3)$ symmetry in QCD.

Thus, the lore is:

In the low temperature phase $Z(3)$ symmetry signals the confinement of colour charges and in the high temperature phase spontaneous breakdown of $Z(3)$ symmetry means colour charges screened by gluon fields and the Polyakov line is an appropriate order parameter. In QCD with dynamical quarks, $Z(3)$ symmetry is explicitly broken and Polyakov loop ceases to be the appropriate order parameter. It should be stressed that these results reflect the grand canonical ensemble treatment of full QCD with respect to triality.

These results are grounds for nonperturbative MC analysis of QCD providing a theoretical basis of quark-gluon plasma phenomenology. However, this treatment is not complete and absolutely satisfactory. There exist several reasons to believe in such a conclusion. This approach does not solve the long standing problem of an order parameter for full QCD. Moreover, it fails in tempting to treat $Z(3)$ metastable phases as physical states in Minkowski space due to running into thermodynamical inconsistency. We shall argue that this picture is conceptually not perfect as well, since it does not substantiate unified mechanism of quark confinement at zero and finite temperature. Indeed, confinement at zero temperature implies only zero triality states and unbroken $Z(3)$ symmetry which triality is connected with. Meanwhile, the corresponding symmetry is explicitly broken in full QCD at any low temperature. At least, it does not answer the important question what occurs if the triality charge is not screened and develops - according to the appropriate Gauss law - a long-range observable which could be, in principle, experimentally measurable.

We are going to demonstrate below that there are no such problems in the canonical ensemble description of finite temperature QCD.

2 $Z(3)$ symmetry in the canonical ensemble description (CE). Euclidean formulation

We develop a view [1, 3, 4] different from that mentioned above which seems more consistent at least in many general aspects. Let us begin mentioning the Euclidean formulation of the theory at zero temperature. The Lagrangian of QCD is symmetric under transformations of the gauge colour group $SU(3)_{loc}$. There is a global subgroup of the local symmetry which can be decomposed as $SU(3)_{gl} = SU(3)/Z(3)_{gl} \otimes Z(3)_{gl}$. Gauge fields are identically invariant under $Z(3)_{gl}$ transformations but fermionic fields transform as

$$\Psi(x) \rightarrow e^{i\frac{2\pi}{3}k}\Psi(x) = e^{i\frac{2\pi}{\sqrt{3}}\lambda^8 k}\Psi(x) \tag{9}$$

leaving the QCD Lagrangian invariant. This symmetry is related to triality t_q which is defined as

$$t_q = (N_q - N_{\bar{q}}) mod\ 3 \tag{10}$$

or as expectation value of the operator

$$t_q =< \sum_x \bar{\Psi}(x)\sqrt{3}\lambda^8\Psi(x) > . \tag{11}$$

It had been proposed [17] that in QCD the gluon colour charges and any zero triality charges are screened by a dynamical Higgs mechanism (understood as a screening mechanism) leaving $Z(3)$ symmetry unbroken and a single quark (having nonzero triality) unscreened. Then long-range chromoelectric forces between quarks will persist according to the Gauss theorem for a discrete centre.

In the scheme described in the previous section (GCE) a single quark may have a finite free energy and the states with open triality can contribute to the partition function. As a result the domains with different $Z(3)$ phases give different contributions to the partition function. The zero phase contributes most. We are sticking here to the opinion that this problem is an artifact of the GCE description which leads to the explicit violation of the $Z(3)$ aperiodic symmetry. We shall show below that these aperiodic transformations at finite temperature can be regarded as acting on links in time direction or equivalently on fermionic fields. These finite temperature transformations of fermionic fields must be treated in analogy to the zero temperature transformations (9). They are related to triality in the same manner as (9). Since our premise is the statement that QCD with dynamical fermions is able to describe only systems of zero triality as it is at zero temperature and all quark phases should be indistinguishable both in the partition function and in thermodynamical functions we would like to construct a theory which is invariant under the corresponding $Z(3)$ transformations.

Let us first describe how one can construct such a theory for pure gluodynamics. In the absence of external perturbation (like "magnetic" field for spin systems), the average value of the Polyakov loop will be equal to zero at any temperature even in the thermodynamical limit: as $\int \mathcal{D}[U]\ e^{-S_G[U]}$ is $Z(3)$ invariant and the Polyakov loop (1) is not, one has to get

$$< L >= 0 \tag{12}$$

in the confined and also in the deconfined phase. The important point is there is no contradiction between this statement and the spontaneous breaking of $Z(3)$ symmetry in the deconfined phase. To demonstrate that one can use

$< L(0)L^*(\vec{r}) >$ correlations or even more suitable the ($Z(3)$ symmetric) $L(\vec{r})$ distributions expressed by the moments $|L(\vec{r})|^2, |L(\vec{r})|^3, \cdots$, what is equivalent to the determination of Polyakov loops in $Z(3)$ invariant representations (octet, decuplet, antidecuplet, etc) [18].

An obstacle for accepting $< L >= 0$ in the deconfined phase comes from lattice calculations which give $< L >\neq 0$ at high β. This result has to be compared with the usual method to demonstrate spontaneous symmetry breaking by an external field. Including $-\lambda \sum_{\vec{r}} Re\{zL(\vec{r})\}, z \in \{1, e^{\pm 2\pi i/3}\}$ in the QCD action, performing the thermodynamic limit and then the limit to vanishing λ would result in $< zL >\neq 0$ and positive in the deconfined phase. As we cannot influence a physical system by a field which we have switched off we have to give an interpretation of this result: The external field fixes the coordinate system in which the "direction" of spontaneous symmetry breaking is measured. This direction is unchanged even after switching off the external field. To get $< L >= 0$ we have to use the "unbiased" path integral which averages over all directions of such a coordinate system and gives $< L >= 0$ also in the deconfined phase. The observation that for a $Z(3)$ invariant Lagrangian the observables of nonvanishing triality have expectation value zero is rather trivial for the pure gluonic case (see [1, 8] for more details). Nevertheless, the usage of the "unbiased" path integral (i.e., with perturbation and averaging as described above) is more preferable since we do not have violation of triality in such a treatment despite deconfinement takes place. We will now turn to the more interesting case of full QCD.

Periodic boundary conditions in imaginary time direction imply that we are dealing with a theory defined in a space with topology $R^3 \otimes S^1$. To construct a theory with conserved triality and indistinguishable phases of quark fields we will follow a recently developed ideology [11, 13] (see also [12, 14] and references therein). Fields of gauge theories on space-time manifolds that are not simply connected need not to be strictly periodic on noncontractible closed loops of this manifold. Instead, these fields need only be periodic up to the action of an element of $Z(3)$. Just these boundary conditions distinguish local $Z(3)$ symmetry from global $Z(3)$ symmetry and allow the local symmetry to manifest itself through a nontrivial Aharonov-Bohm effect.

Let us consider the Taylor expansion of $\exp(-S_F)$ in the path integral. The first nontrivial term, coming from time-like links, which will survive the integration over fermionic fields is proportional to $\prod_t \bar{\Psi}_t \Psi_t U_0(t)$. This term describes propagation of a single quark along the temperature direction. The

triality transformations of the fermionic fields in $\prod_t \bar{\Psi}_t \Psi_t U_0(t)$ should give a nontrivial phase factor. It is obvious that one can achieve this by considering in addition to (9) the following substitutions for the fermionic fields

$$\Psi(\vec{x}, \beta) \to -e^{i\frac{2\pi}{3}k} \Psi(\vec{x}, 0). \tag{13}$$

If we now perform this substitution in the operator $\prod_t \bar{\Psi}_t \Psi_t U_0(t)$ and then consider the action of the transformation (9) on this operator we will find that the operator gets a nontrivial phase factor as it should be for a state with a single quark. Thereby, this is the obvious extension of triality transformations (9) to the finite temperature theory. The phase transformation (9) determines the triality of quark states at zero temperature. Hence, this transformation together with (13) determines the triality of quark states at finite temperature.

In the zero temperature theory the transformation (9) acts trivially on all physical states. We would like to construct a formulation of QCD where the corresponding finite temperature transformations act trivially on physical states as well. To get such a theory where only triality zero states contribute we have to sum over the following boundary conditions in the time direction

$$\Psi(\vec{x}, \beta) = -e^{i\frac{2\pi}{3}k} \Psi(\vec{x}, 0). \tag{14}$$

This is equivalent to a summation over all phases in (9) combined with (13) and to a projection onto zero triality states.

We explain now that such a projection is nothing but the requirement of local gauge invariance in the meaning of refs. [12] and [13]. The boundary conditions (14) allow to identify field configurations which are related by the transformation (13). Such a relation can be considered as the definition of a local discrete symmetry [12] and is known as the so-called orbifold construction. It was shown [12] that one must project out noninvariant states from the theory with local gauge symmetry of such a type. Thus, because of the identification (14) all physical observables should be invariant under transformation of the local $Z(3)$ group. All noninvariant states, like single quark states, must be projected out from the theory to obtain a true Hilbert space. This defines an ensemble canonical with respect to triality. On the other hand, it is a natural extension of the orbifold construction for nonabelian gauge theories at finite temperature.

There is no difficulty to demonstrate that the summation over all boundary conditions (14) is identical with the projection onto the zero triality sector of the fermionic determinant (a similar proof that only quark loops of zero triality contribute to the partition function in this case has also been given in [3]). We can define the triality operator $\hat{\mathcal{T}}$ (with eigenvalues $0, \pm 1$) which determines the triality of closed loops, and with his help a projection operator $\hat{P}_{\mathcal{T}}$ on triality $\mathcal{T}$

$$\hat{P}_{\mathcal{T}} = \frac{1}{3} \sum_{k=0,\pm 1} e^{k 2\pi i (\hat{\mathcal{T}} - \mathcal{T})/3}. \tag{15}$$

The implementation of this projection operator in the lattice formulation is very simple. One has to multiply all links in time direction for an arbitrary time step with the phase $e^{k2\pi i/3}$ and to determine with this new link variables the determinant. The result has to be multiplied with $e^{k2\pi i \mathcal{T}/3}$. Finally, the sum over phases $k = 0, \pm 1$ has to be executed. Then the triality zero contribution of the fermionic determinant is

$$\det{}_0 M = \hat{P}_0 \det M = \frac{1}{3} \sum_{k=0,\pm 1} e^{k 2\pi i \hat{\mathcal{T}}/3} \det M \tag{16}$$

and the full fermionic determinant is the sum over all triality sectors

$$\det M = \sum_{\mathcal{T}=0,\pm 1} \det{}_{\mathcal{T}} M, \tag{17}$$

with

$$\det{}_{\mathcal{T}} M = \hat{P}_{\mathcal{T}} \det M. \tag{18}$$

It can be easily seen that this procedure is exactly equivalent to the summation over all boundary conditions (14). From eq. (17) it is obvious that the observable L having triality $\mathcal{T} = 1$ tests only the $\mathcal{T} = -1$ sector of the fermionic determinant which is not the vacuum sector. Using the correct $\mathcal{T} = 0$ vacuum sector for the fermionic determinant results in a $Z(3)$ symmetric L-distribution. Let us discuss this in more detail. As the measure $\mathcal{D}[U]$ and the gluonic Lagrangian $S_G[U]$ are $Z(3)$ symmetric, the product $\det M[U] O_{\mathcal{T}}[U]$ in

$$< O_{\mathcal{T}} >= \frac{1}{Z} \int \mathcal{D}[U] e^{-S_G[U]} \det M[U] O_{\mathcal{T}}[U] \tag{19}$$

has to be also $Z(3)$ symmetric to get $< O_{\mathcal{T}} > \neq 0$. In other words $Z(3)$ violating contributions in $\det M[U] O_{\mathcal{T}}[U]$ are automatically eliminated by the path integral. Only the triality $-\mathcal{T}$ component $\det_{-\mathcal{T}} M$ of the fermionic determinant survives. $\det_{-\mathcal{T}} M$ has $\mathcal{T}$ quark loops less than antiquark loops winding around the lattice in time direction.

Applying this statement to the example of the single Polyakov loop L representing an infinitely heavy quark Q, the fermionic determinant supplies a triality -1 state of light quarks which will mostly consist of a light antiquark $\bar{q}$ in order to colour neutralize the heavy quark Q. The Polyakov expectation value $< L >$ is a thermodynamical mixture

$$< L > = ... + e^{-F(Q\bar{q})/T} + e^{-F(Qqq)/T} + ... \tag{20}$$

of a heavy-light meson $Q\bar{q}$, a heavy-light baryon Qqq, ... These heavy-light states should be bound in the hadronic phase and "ionized" in the quark-gluon plasma phase. The charge density of such light quarks around static sources has been measured in ref. [19].

For the system of a heavy quark Q at position $\vec{r}_1$ and a heavy antiquark $\bar{Q}$ at position $\vec{r}_2$

$$L(\vec{r}_1) L^*(\vec{r}_2) \tag{21}$$

a totaly different component of the fermionic determinant, the triality zero component, contributes. It is evident that this component is $Z(3)$ symmetric, i.e. the three $Z(3)$ transformed sectors which differ in L by factors $e^{\pm\frac{2\pi i}{3}}$ contribute with the same Boltzmann factor. This example shows that for observables of triality zero full QCD is $Z(3)$ symmetric. The fermionic action does not destroy $Z(3)$ symmetry if we choose to work in the present scheme.

3 $Z(3)$ symmetry in the Hamiltonian formulation

It may be instructive to see how the $Z(3)$ symmetry enters the Hamiltonian formulation of QCD. In temporal gauge we have to integrate over all time-independent gauge transformations of gauge and of matter fields. This defines a projection operator

$$P = \int D[\alpha_{\vec{x}}] \exp[-i \int d\vec{x} \alpha^a(\vec{x})((DE)^a - \rho^a) + i \int_s d\vec{s} \alpha^a E^a], \tag{22}$$

where $\rho^a = \bar{\Psi}_{\vec{x}}(\lambda^a/2)\Psi_{\vec{x}}$. P projects onto "physical" states which have to fulfil a local Gauss law: $(DE)^a = \rho^a$ and a global one: $\int_V divE^a dV = Q^a = 0$. The global Gauss law warrants that physical states are global colour singlets. We would like to concentrate here on the centre of the global gauge transformations. In this case the operator (22) takes the form

$$P = \delta((DE)^a - \rho^a)P_{tr} \tag{23}$$

which includes a Kronecker-delta for the quark triality charge

$$P_{tr} = \frac{1}{N}\sum_{k=0}^{k=N-1} \exp[\frac{2\pi ik}{N}(\sum_x \bar{\Psi}_x\lambda^a\Psi_x)] \tag{24}$$

(where $\lambda^a = \sqrt{3}\lambda^8$ for $SU(3)$) and, thus, defines canonical ensemble with respect to triality. Following the usual procedure [20] one can return to the Lagrangian formulation with a summation over all boundary conditions (14) for fermionic fields.

Instead of the canonical ensemble with $B = \mathcal{T} = 0$ which should be well suited, a grand canonical description is allowed if it predicts the same behaviour in the thermodynamical limit. Common lattice Monte-Carlo calculations in full QCD use the full fermionic determinant $\det M$ and thus simulate a grand canonical ensemble with chemical potential $\mu = 0$ containing all three triality sectors. We want to point out that there are neither requirements nor conditions in the theory of strong interactions with long-range gauge fields guaranteeing that these two descriptions should coincide [21, 22]. One of the main conclusion of ref. [21] is that one must verify the agreement between ensembles in each particular case (i.e., for each conserved quantum number) separately. It has been proved [21] for free theories that canonical and grand canonical ensembles are identical in the thermodynamical limit with respect to the baryonic number and the strangeness. A similar proof for the global colour charge has been done in ref.[23]. We have shown here that for the full theory grand canonical and canonical ensembles differ with respect to quark triality: the $Z(3)$ symmetry has different realizations in these two ensembles. In ref.[4] it was demonstrated that some of thermodynamical functions may have different behaviour as well.

The most often objection against the above picture is that the projection onto the zero triality sector defines a new theory which can somehow differ from the genuine QCD. We believe that the present consideration avoids

this misunderstanding, which has nothing to do with the discussed situation. We have shown that this projection is equivalent to using the canonical ensemble with respect to triality in the Hamiltonian formulation. Further, the Lagrangian formulation of the theory showed that we did not modify the QCD action. The canonical ensemble description is realized in this formulation by means of a summation over the corresponding boundary conditions.

4 Phase structure of full QCD

The present consideration allows us to reexamine a possible phase structure of full QCD and to speculate about the high temperature region within the canonical ensemble description. One may claim by now that a quantum field system with a nonabelian local gauge symmetry including discrete (local and/or global) $Z(N)$ symmetry can be in one of the following states depending on the concrete representations of gauge and matter fields in the initial Lagrangian:

1. Coulomb phase, gauge fields are massless.

2. Higgs phase of the first type, triality screening phase. Gauge symmetry is completely broken, gauge fields acquire masses.

3. Higgs phase of the second type, all colour charges excepting nonzero triality are screened. Gauge symmetry is broken up to its discrete centre, no massless gauge fields. Triality charges are screened classically but may be detected at long distances by specific quantum mechanical process.

4. Confinement phase. Triality charges are unscreened in any sense but cannot be detected at long distances. Gauge symmetry is broken either up to its $Z(N)$ subgroup (vortex mechanism of confinement) or up to its abelian subgroup (monopole mechanism).

We would like to conjecture the following scenario to be realized in QCD with dynamical quarks in the fundamental representation:

1. QCD at zero temperature always is in the confining phase. It seems this statement is generally accepted by now although there is no strict proof of this fact.

2. Full QCD at finite temperature, as described in the grand canonical ensemble, always is in the Higgs phase of the first type where also triality charges are screened. The aperiodic $Z(3)$ symmetry is explicitly broken at any temperature and domains with different phases contribute in different

ways to the partition function. In the thermodynamical limit only one domain with fixed phase ($k = 0$ in the commonly used description) can be found.

3. Full QCD at finite temperature, as described in the canonical ensemble, is in the confining phase at low temperature. When the temperature increases a phase transition to the Higgs phase of the second type takes place. To decide what phase is realized in both ensembles one can apply an order parameter proposed in [13]. This order parameter can probe a phase of a matter field which carries $Z(N)$ charge. In the GCE only one domain can be found in the thermodynamical limit. It implies that the corresponding order parameter will be always equal to 1 ($k = 0$ phase). In the CE all domains with different phases are degenerate (see discussion just below) and we should expect that this order parameter can distinguish between different phases in the thermodynamical limit. This means that triality charges of quarks can be detected at long distances via the Aharonov-Bohm effect [11, 13].

One of the interesting consequences of the canonical description concerns the possible states of quark-gluon matter at high temperature. Recently, in refs. [24, 25] it has been concluded on the one hand that explicit $Z(3)$ symmetry breaking leads to the existence of metastable states at arbitrary high temperatures. On the other hand the authors of refs. [26, 27] believe that $Z(3)$ phases cannot be prepared as real macroscopic systems [28]. These models are based on the $Z(3)$ asymmetric L-distribution in the quark-grand canonical ensemble. But as we argued above one has to investigate the zero-triality sector of the fermionic determinant in order to study the QCD vacuum. In the corresponding ensemble one obtains $Z(3)$ symmetric L-distributions and hence no metastable phases can be found which persist up to infinitely high temperatures. The effective models of refs. [24, 25, 26, 27] are effective models of heavy-light hadrons in the QCD vacuum and not effective models of "the" QCD vacuum. Exact two-loop calculations in CE, which demonstrate that all $Z(N)$ phases are left degenerate in CE, have recently been presented in [29]. It is just this degeneracy which gives the possibility to observe different $Z(N)$ domains via the Aharonov-Bohm effect.

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No.930224.

References

[1] M. Faber, O. Borisenko, S. Mashkevich, G. Zinovjev, Fresh look on triality, IK-TUW-Preprint 9209401 (1992) and 9308401 (1993);
Triality in QCD, Proc. of Int. School-Seminar'93 "Hadrons and Nuclei from QCD" (ed.by K.Fyjii, Y.Akaishi, B.L.Reznik. World Scientific Singapore-New-Jersey-London-Hong-Kong) p.333.

[2] Triality and the grand canonical ensemble in QCD, M. Faber, O. Borisenko, S. Mashkevich, G. Zinovjev, Proc. of "Lattice'94", Nucl. Phys. B (Proc. Suppl.), in print.

[3] M. Oleszczuk and J. Polonyi, Canonical vs. grand canonical ensemble in QCD, TPR 92-34, 1992.

[4] M. Oleszczuk and J. Polonyi, Ann. Phys. 227 (1993) 76.

[5] A.V. Smilga, Ann. Phys. 234 (1994) 1.

[6] Ian I. Kogan, Phys. Rev. D 49 (1994) 6799.

[7] T.H. Hansson, H.B. Nielsen and I. Zahed, QED with unequal charges, a study of spontaneous Z_n symmetry breaking, Preprint USITP-94-09, SUNY-NTG-94-23, 1994.

[8] J. Kiskis, Phase of the Wilson line, Preprint UCD 94-21, 1994.

[9] J. Boorstein, D. Kutasov, Wilson loops, winding modes and domain walls in finite temperature QCD, Preprint EFI-94-42, HEP-TH-9409128.

[10] L. D. McLerran and B. Svetitsky, Phys. Rev. D 24 (1981) 450; B. Svetitsky and L.G. Yaffe, Nucl. Phys. B 210 (1982) 423; J. Kuti, J. Polonyi, K. Szlachanyi, Phys. Lett. 98 B (1981) 199

[11] L.M. Krauss and F. Wilczek, Phys. Rev. Lett. 62 (1989) 1221; T. Banks, Nucl. Phys. B 323 (1989) 90.

[12] K. Li, Nucl. Phys. B 361 (1991) 437.

[13] J. Preskill, L.M. Krauss, Nucl. Phys. B 341 (1990) 50.

[14] S. Coleman, J. Preskill, F. Wilczek, Nucl. Phys. B 378 (1992) 175.

[15] J. B. Kogut, Rev. Mod. Phys. 55 (1983) 755.

[16] T. Banks and A. Ukawa, Nucl. Phys. B 225 (1983) 145.

[17] G. Mack, Phys. Lett. B 78 (1978) 263.

[18] P. H. Damgaard, Phys. Lett. B 194 (1987) 107; H. Markum, M. Faber, M. Meinhart, Phys. Rev. D 36 (1987) 632.

[19] W. Bürger, M. Faber, H. Markum and M. Müller, Phys. Rev. D 47 (1993) 3034.

[20] D. Gross, L.G. Yaffe, R.D. Pisarski, Rev. Mod. Phys. 53 (1981) 2305.

[21] R. Hagedorn, K. Redlich, Z. Phys. C 27 (1985) 541 (and references therein).

[22] C. DeTar and L. D. Mc Lerran, Phys. Lett. B 119 (1982) 171.

[23] M. Gorenstein, S. Lipskikh, V. Petrov, G. Zinovjev, Phys.Lett. B 123 (1983) 437; H.-Th Elze, W. Greiner, J. Rafelski, Phys.Lett. B 124 (1983) 515.

[24] V. Dixit and M. C. Ogilvie, Phys. Lett. B 269 (1991) 353.

[25] J. Ignatius, K. Kajantie and K. Rummukainen, Phys. Rev. Lett. 68 (1992) 737.

[26] V. M. Belyaev, Ian I. Kogan, G. W. Semenoff and N. Weiss, Phys. Lett. B 277 (1992) 331.

[27] W. Chen, M. I. Dobroliubov and G. B. Semenoff, Phys. Rev. D 46 (1992) R1223.

[28] For a discussion concerning this point see also A. Gocksch and R. D. Pisarski, Nucl. Phys. B 402 (1993) 657.

[29] M. Oleszczuk, Degenerate $Z(2)$ domains in the $SU(2)$ quark canonical ensemble, TPR-94-11, 1994.